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STUDY OF THE DYNAMICS OF A SYNCHRONOUS MOTOR BY ASYMPTOTIC METHODS

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We investigate the complete system of differential equations describing the dynamics of a synchronous motor with two windings on the rotor, under the assumption that the moment of inertia of the rotor is sufficiently large. We consider two domains of variation of the variable s defining the rotor slippage. In one of them s have finite values, while in the other domain s are small. In the first case we investigate the solutions of the complete system of equations periodic in θ, and in the second case we study the periodic solutions which embrace the state of equilibrium. The conditions of stability of the solutions obtained are given. The stable periodic solutions correspond in the first case to the synchronous modes of the synchronous motor, and in the second case to the oscillations of the rotor relative to the synchronous rate of rotation.

When the transient processes in a synchronous motor are investigated using the complete system of differential equations obtained by Gorev in [1], the following approaches are usually employed: (1) only the equation of the mechanical motion of the rotor is considered [2 – 7]; (2) only the electrical equations are considered, i. e. the transient processes are considered at a constant angular velocity of rotation of the rotor; (3) the complete system of equations is linearized near the steady state motion and small oscillations of the system are studied; (4) the complete system of equations is integrated numerically [1, 8]. However, the dynamics of a synchronous motor as such, has not been investigated to any great extent.

1. The equations of dynamics and statement of the problem. The equations of dynamics of a synchronous motor working in parallel with a network of infinite power, in the driving mode, assume the following form [1] after introducing the dimensionless variables, the parameters and time $\tau = \omega_1 t$:

$$x^{\cdot} + \xi x + \lambda(1-s)\,y + \gamma_1 u^{\cdot} + \gamma_2(1-s)\,v = -\sin\theta \qquad (1.1)$$
$$-(1-s)\,x + \lambda y^{\cdot} + \xi y - \gamma_1(1-s)\,u + \gamma_2 v^{\cdot} = -\cos\theta$$
$$\gamma_1 x^{\cdot} + \alpha_1 u^{\cdot} + \beta_1 u = \eta, \quad \gamma_2 y^{\cdot} + \alpha_2 v^{\cdot} + \beta_2 v = 0$$
$$\theta^{\cdot} = s, \quad s^{\cdot} = M_0\,\{T_0 - [\gamma_2 xv - \gamma_1 yu - (1-\lambda)\,xy]\}$$
$$\alpha_1 = \frac{2}{3}\frac{L_4}{L_x}, \quad \beta_1 = \frac{2}{3}\frac{R_4}{\omega_1 L_x}, \quad \gamma_1 = \frac{M_x}{L_x}$$
$$\alpha_2 = \frac{2}{3}\frac{L_5}{L_x}, \quad \beta_2 = \frac{2}{3}\frac{R_5}{\omega_1 L_x}, \quad \gamma_2 = \frac{M_y}{L_x}$$
$$\lambda = \frac{L_y}{L_x}, \quad \xi = \frac{R}{\omega_1 L_x}, \quad \eta = \frac{2}{3}\frac{E_4}{E_0}, \quad M_0 = \frac{3}{2}\frac{E_0^2}{J\omega_1^4 L_x}, \quad T_0 = \frac{3}{2}\frac{L_x T\omega_1^2}{E_0^2}$$

Here θ is the angle between the transverse axis of the rotor and the rotating emf (electromotive force) vector of the external network; x, y, u and v are the reduced longitudinal and transverse stator and rotor currents; s is the rotor slippage; ω_1 is the frequency of the external network; L_x and L_y are the coefficients of self-induction in the longitudinal and transverse axes of the motor; L_4 and L_5 are the coefficients of self-induction of the rotor windings; M_x and M_y are the coefficients of mutual induction of the stator phases with the rotor windings; R, R_4 and R_5 denote the active resistances of the stator phases and the rotor windings; E_0 is the amplitude of the external network potential; E_4 is the excitation emf; J is the moment of inertia of the rotor and T is the moment of the external mechanical forces applied to the rotor shaft.

When the synchronous motor is in its operational (synchronous) mode, the rotor slippage $s = 0$. Nevertheless, the variable s may assume any value during the transient modes, e.g. starting the motor or sharp variations of the load. Let the time constant of the mechanical motion be much greater than the greatest time constant of the electrical loops by virtue of the fact that the moment of inertia of the rotor J is sufficiently large, i.e. the parameter $M_0 \equiv \varepsilon \ll 1$ (this implies another simultaneous assumption that the greatest time constant of the electrical loops is of the order of unity). Using the above assumption, we study the dynamics of a synchronous motor by means of asymptotic methods, for the finite values of s and for small s.

2. Investigation of the dynamics in the domain of finite values of s. Let us first consider the dynamics of a synchronous motor in the domain of asynchronous modes when the rotor slippage s, is different from zero and is not small ($|s| \sim 1$). The system (1.1) can be written in the form

$$\theta^{\cdot} = s, \quad z^{\cdot} = A(s)\,z + a(\theta), \quad s^{\cdot} = \varepsilon F(z) \quad (z \equiv \operatorname{col}\{x, y, u, v\}) \qquad (2.1)$$

where z and θ are rapidly varying variables and, physically speaking, θ is a rapidly rotating phase. The eigenvalues of the matrix $A(s)$ have negative real parts when $\alpha_1 - \gamma_1^2 > 0$ and $\lambda\alpha_2 - \gamma_2^2 > 0$ and this takes place at all times since the left hand sides of these inequalities represent the longitudinal and transverse leakage coefficients of the motor windings [1].

We shall seek a solution of (2.1) which is near to the steady state solution of the degenerate system obtained from (2.1) for $\varepsilon = 0$. Denoting the solution of the degenerate system by $z_0(s, \theta)$ we perform the following change of variables in (2.1):

$$z = z_0(s, \theta) + \varepsilon\sqrt{\varepsilon}\,\zeta + \varepsilon z^\circ(\theta) \tag{2.2}$$
$$z_0(s, \theta) \equiv \{x(s, \theta),\ y(s, \theta),\ u(s, \theta),\ v(s, \theta)\}$$

where ζ is a new variable and the vector function $z^\circ(\theta)$ will be defined below. Then the following expansion of the function $F(z)$ near the solution $z_0(s, \theta)$ holds:

$$F(z) = F(z_0(s,\theta)) + \varepsilon(\sqrt{\varepsilon}\zeta + z^\circ(\theta))\frac{\partial F}{\partial z}(z_0) + \varepsilon^2(\ .\ .\ .\ ) =$$
$$\langle F(s)\rangle + F^*(s,\theta) + \varepsilon(\sqrt{\varepsilon}\zeta + z^\circ(\theta))\Big(\frac{\partial\langle F\rangle}{\partial z}(s) +$$
$$\frac{\partial F^*}{\partial z}(s,\theta)\Big) + \varepsilon^2(\ .\ .\ .\ )$$

where the mean values of $F^*(s, \theta)$ and $\partial F^*(s, \theta)/\partial z$ averaged over the variable θ are zero.

Let s_0 be the root of the equation $\langle F(s_0)\rangle = 0$. We perform the following substitution [9]:

$$s = s_0 + \varepsilon w + \frac{\varepsilon}{s_0}\int_{\theta_0}^{\theta} F^*(s_0, \theta)d\theta + \varepsilon s^\circ \tag{2.3}$$

where w is a new variable and s° will be defined below. Then the system (2.1) assumes the form

$$\theta^{\cdot} = s_0 + \varepsilon w + \frac{\varepsilon}{s_0}\int_{\theta_0}^{\theta} F^*(s_0,\theta)d\theta + \varepsilon s^\circ \tag{2.4}$$
$$w^{\cdot} = \varepsilon\frac{d\langle F\rangle}{ds}w + \varepsilon\frac{\partial\langle F\rangle}{\partial s}s^\circ + \varepsilon\frac{\partial\langle F\rangle}{\partial z}\langle z^\circ\rangle + \frac{\varepsilon}{s_0}\frac{\partial F^*}{\partial s}(s_0,\theta)\int F^* d\theta +$$
$$\varepsilon\frac{\partial F^*}{\partial z}(s_0\theta)z^{\circ *}(\theta) - \frac{\varepsilon}{s_0^2}F^*(s_0,\theta)\int F^* d\theta + \varepsilon\frac{\partial\langle F\rangle}{\partial z}z^{\circ *}(\theta) +$$
$$\varepsilon\frac{\partial F^*}{\partial z}(s_0,\theta)\langle z^\circ\rangle + \frac{\varepsilon}{s_0}\frac{d\langle F\rangle}{ds}\int F^* d\theta + \varepsilon\frac{\partial F^*}{\partial s}w + \varepsilon\frac{\partial F^*}{\partial s}s^\circ +$$
$$\varepsilon\sqrt{\varepsilon}\frac{\partial F^*}{\partial z}\zeta - \frac{\varepsilon}{s_0}F^* w - \frac{\varepsilon}{s_0}F^*(s_0,\theta)s^\circ + \varepsilon\sqrt{\varepsilon}\frac{\partial\langle F\rangle}{\partial z}\zeta +$$
$$\varepsilon^2(\ .\ .\ .\ )$$
$$\sqrt{\varepsilon}\zeta^{\cdot} + \frac{dz^\circ}{d\theta} = \sqrt{\varepsilon}A(s_0)\zeta + A(s_0)z^\circ - z_{0s}'(s_0,\theta)F^*(s_0,\theta) + \varepsilon(\ .\ .\ .\ )$$

Let us assume that $d\langle F(s_0)\rangle / ds \neq 0$. We choose the vector function $z^\circ(\theta) = \langle z^\circ\rangle + z^{\circ *}(\theta)$ (the mean value of $z^{\circ *}(\theta)$ averaged over θ is zero) as a particular solution of the inhomogeneous system of linear differential equations with constant coefficients

$$\frac{dz^\circ}{d\theta} = A(s_0)z^\circ - z_0s'(s_0,\theta)F^*(s_0,\theta)$$

and s° as the solution of the linear equation

$$\frac{d\langle F\rangle}{ds}s^\circ + \frac{\partial\langle F\rangle}{\partial z}\langle z^\circ\rangle + \langle W(s_0)\rangle = 0$$

where $\langle W(s_0)\rangle$ is the mean value of

$$W(s_0, \theta) = \frac{1}{s_0}\frac{\partial F^*}{\partial s}\int F^*(s_0, \theta)\, d\theta + \frac{\partial F^*}{\partial z} z^{\circ *}(\theta) - \frac{1}{s_0^2} F^* \int F^*(s_0, \theta)\, d\theta$$

averaged over θ. Then the system (2.3) is reduced to

$$\theta^{\cdot} = s_0 + \Theta(\theta, w, \zeta, \varepsilon) + \varepsilon\Theta^*(\theta, w, \zeta, \varepsilon) \tag{2.5}$$

$$w^{\cdot} = \varepsilon \frac{d\langle F\rangle}{ds} w + \varepsilon X(\theta, w, \zeta, \varepsilon) + \varepsilon X^*(\theta, w, \zeta, \varepsilon)$$

$$\zeta^{\cdot} = A(s_0)\zeta + Y(\theta, w, \zeta, \varepsilon)$$

here

$$\Theta(\theta, w, \zeta, \varepsilon) = \varepsilon(w + s^{\circ}), \quad \Theta^*(\theta, w, \zeta, \varepsilon) = \frac{1}{s_0}\int_{\theta_0}^{\theta} F^*(s_0, \theta)\, d\theta$$

$$X(\theta, w, \zeta, \varepsilon) = \sqrt{\varepsilon}\frac{\partial\langle F\rangle}{\partial z}\zeta + \varepsilon(\ldots)$$

$$X^*(\theta, w, \zeta, \varepsilon) = \frac{\partial\langle F\rangle}{\partial z} z^{\circ *}(\theta) + \frac{\partial F^*}{\partial z}(s_0, \theta)\langle z^{\circ}\rangle + \frac{1}{s_0}\frac{d\langle F\rangle}{ds}\int F^* d\theta +$$
$$\frac{\partial F^*}{\partial s}(s_0, \theta) w + \frac{\partial F^*}{\partial s}(s_0, \theta) s^{\circ} + \sqrt{\varepsilon}\frac{\partial F^*}{\partial z}(s_0, \theta)\zeta - \frac{1}{s_0} F^*(s_0, \theta) w -$$
$$\frac{1}{s_0} F^*(s_0, \theta) s^{\circ} + W(s_0, \theta) - \langle W(s_0)\rangle + \varepsilon(\ldots)$$

$$Y(\theta, w, \zeta, \varepsilon) = \sqrt{\varepsilon}(\ldots)$$

Clearly the mean values of the functions $\Theta^*(\theta, w, \zeta, \varepsilon)$ and $X^*(\theta, w, \zeta, \varepsilon)$ averaged over θ are equal to zero. Thus the system (2.5) satisfies the conditions of the theorems 15.1, 15.3 and 15.4 of Hale [10]. According to these theorems the system (2.1) has a one-dimensional integral manifold

$$s = s_0 + \varepsilon s^{\circ} + \frac{\varepsilon}{s_0}\int_{\theta_0}^{\theta} F^*(s_0, \theta)\, d\theta + \varepsilon f(\theta, \varepsilon) \tag{2.6}$$

$$z = z_0(s_0, \theta) + \varepsilon z^{\circ}(\theta) + \varepsilon\sqrt{\varepsilon} g(\theta, \varepsilon)$$

$$f(\theta, \varepsilon) \to 0, \qquad g(\theta, \varepsilon) \to 0 \quad \text{as} \quad \varepsilon \to 0$$

The integral manifold (2.6) is a θ-periodic solution of (1.1) near the steady state solution $s = s_0$, $z = z_0(s_0, \theta)$ of the degenerate system. The periodic solution is stable when $d\langle F(s_0)\rangle / ds < 0$ and is a saddle when $d\langle F(s_0)\rangle / ds > 0$

For the initial system (1.1) the solution $z_0(s_0, \theta) \equiv \{z_{0j}(s_0, \theta)\}$ has the form

$$z_{0j}(s_0, \theta) = \frac{(-1)^j}{a^2 + b^2}\{\sin\theta[a(a_{1j} + b_{2j}) + b(b_{1j} - a_{2j})] +$$
$$\cos\theta[a(b_{1j} - a_{2j}) - b(a_{1j} + b_{2j})]\} - (-1)^j \eta\frac{A_{3j}(0)}{\Delta(0)}, \qquad j = 1, 2, 3, 4$$

$$a = \operatorname{Re}\Delta(p), \ b = \operatorname{Im}\Delta(p), \ a_{kj} = \operatorname{Re}A_{kj}(p), \ b_{kj} = \operatorname{Im}A_{kj}(p)$$
$$(p = is, \ i = \sqrt{-1})$$

Here $\Delta(p)$ is the determinant of the system (1.1), $A_{kj}(p)$ are the minors of $\Delta(p)$ and

$$F(z_0) = F(s_0, \theta) = T_0 - a_1(s_0)\eta^2 - b_1(s_0) - b_2(s_0)\cos 2\theta -$$
$$b_3(s_0) \times \sin 2\theta - \eta c_1(s_0)\cos\theta - \eta c_2(s_0)\sin\theta$$

where s_0 is the root of the equation

$$\langle F(s_0)\rangle \equiv T_0 - a_1(s_0)\eta^2 - b_1(s_0) = 0$$

$$a_1(s_0) = -\frac{\gamma_1^2\xi(1-s_0)}{\beta_1^2}\frac{\xi^2+\lambda^2(1-s_0)^2}{[\xi^2+\lambda(1-s_0)^2]^2}$$

$$b_1(s_0) = -\frac{1}{2(a^2+b^2)}\{\gamma_2[(a_{11}+b_{21})(a_{14}+b_{24})+(b_{11}-a_{21})(b_{14}-a_{24})] -$$
$$\gamma_1[(a_{12}+b_{22})(a_{13}+b_{23})+(b_{12}-a_{22})(b_{13}-a_{23})] -$$
$$(1-\lambda)[(a_{11}+b_{21})(a_{12}+b_{22})+(b_{11}-a_{21})(b_{12}-a_{22})]\}$$

As an example, let us consider a synchronous motor with the parameters $\alpha_1 = {}^2/_3$, $\alpha_2 = {}^1/_3$, $\beta = {}^{10}/_3$, $\beta_2 = {}^2/_3$, $\gamma_1 = 0.3$, $\gamma_2 = 0.1$, $\lambda = 0.1$, $\eta = {}^{20}/_3$ and $T_0 = 0.3$ and follow the behavior of the periodic solutions relative to the parameter ξ characterizing the ohmic losses in the stator circuit. When $\xi = 0$ the system (1.1) has two θ-periodic solutions, the stable solution (asynchronous mode) $s_{01} = 0.4$ $(d\langle F(s_{01})\rangle / ds = -0.72)$ and the saddle $s_{02} = 21.454$ $(d\langle F(s_{02})\rangle / ds = 0.0134)$. As ξ increases, the coordinates s_{0i} of the periodic solutions also increase, at $\xi = 0.04$ the saddle-type limit cycle $(s_{02} = 24.525)$ begins to decrease with respect to s and from $\xi = 0.71$ $(s_{01} = 0.90578$, $s_{02} = 5.6955)$ both cycles again begin to rise without converging.

3. Investigation of the dynamics in the domain of small s $(|s| \leqslant \varepsilon \ll 1)$. Let us now consider the dynamics of a synchronous motor in the domain of small values of the slippage s $(|s| \leqslant \varepsilon \ll 1)$. We pass to the new variable σ, where $s = \mu\sigma$ and $\mu = \sqrt{\varepsilon}$, introducing "slow" time $\tau' = \mu\tau$. Then the system (1.1) becomes

$$\begin{aligned}
&\mu x^{\cdot} + \xi x + \lambda(1-\mu\sigma)y + \gamma_1\mu u^{\cdot} + \gamma_2(1-\mu\sigma)v = -\sin\theta \\
&-(1-\mu\sigma)x + \lambda\mu y^{\cdot} + \xi y - \gamma_1(1-\mu\sigma)u + \gamma_2\mu v^{\cdot} = -\cos\theta \\
&\gamma_1\mu x^{\cdot} + \alpha_1\mu u^{\cdot} + \beta_1 u = \eta, \qquad \gamma_2\mu y^{\cdot} + \alpha_2\mu v^{\cdot} + \beta_2 v = 0 \\
&\theta^{\cdot} = \sigma, \qquad \sigma^{\cdot} = T_0 - [\gamma_2 xv - \gamma_1 yu - (1-\lambda)xy]
\end{aligned} \tag{3.1}$$

Here and in what follows, the dot denotes differentiation with respect to the slow time τ'. The system (3.1) represents a system with a small parameter accompanying the derivative. Performing the following change of variables:

$$x^\circ = x - \frac{c}{\lambda+\xi^2}, \quad y^\circ = y - \frac{d}{\lambda+\xi^2}, \quad u^\circ = u - \frac{\eta}{\beta_1}, \quad v^\circ = v \tag{3.2}$$

we can reduce (3.1) to the form

$$\mu z^{\cdot} = A(\theta, \sigma, z, \mu)z + \mu F(\theta, \sigma, \mu) \tag{3.3}$$

$$\theta^{\cdot} = \frac{\partial H(\theta,\sigma)}{\partial\sigma} + \mu P_1(\theta,\sigma,\mu) + P_2(\theta,\sigma,z,\mu)z$$

$$\sigma^{\cdot} = -\frac{\partial H(\theta,\sigma)}{\partial\theta} + \mu Q_1(\theta,\sigma,\mu) + Q_2(\theta,\sigma,z,\mu)z$$

where

$$z \equiv \operatorname{col}\{x^\circ, y^\circ, u^\circ, v^\circ\}, \quad b = \gamma_1\eta / \beta_1$$
$$c = -\xi\sin\theta + \lambda\cos\theta - \lambda b, \quad d = -\sin\theta - \xi\cos\theta + \xi b\,.$$

$$A(\theta, \sigma, z, \mu) \equiv A(\mu\sigma) =$$

$$\begin{array}{c} l \\ m \\ l \\ m \end{array} \left\| \begin{array}{cccc} -\alpha_1\xi & -\lambda\alpha_1(1-\mu\sigma) & \gamma_1\beta_1 & -\alpha_1\gamma_2(1-\mu\sigma) \\ \alpha_2(1-\mu\sigma) & -\xi\alpha_2 & \alpha_2\gamma_1(1-\mu\sigma) & \gamma_2\beta_2 \\ \xi\gamma_1 & \lambda\gamma_1(1-\mu\sigma) & -\beta_1 & \gamma_1\gamma_2 \\ -\gamma_2(1-\mu\sigma) & \xi\gamma_2 & -\gamma_1\gamma_2(1-\mu\sigma) & -\lambda\beta_2 \end{array} \right\|$$

$$F(\theta, \sigma, \mu) \equiv F(\theta, \sigma) = \frac{\sigma}{\lambda + \xi^2} \times$$

$$\left\{ \begin{array}{l} l\left[-\lambda\gamma_1^2 \sin\theta + (\alpha_1 - \lambda\alpha_1 - \gamma_1^2)\xi\cos\theta + \lambda\alpha_1\xi b\right] \\ m\left[\xi(\gamma_2^2 + \alpha_2 - \lambda\alpha_2)\sin\theta - \gamma_2^2\cos\theta - \alpha_2\xi^2 b\right] \\ l(-d) \\ mc \end{array} \right.$$

$$P_1 = P_2 = Q_1 = 0$$

$$Q_2(\theta, \sigma, z, \mu) \equiv Q_2(\theta, z) =$$
$$\{-{}^1/_2\gamma_2 v^\circ + {}^1/_2(1-\lambda)y^\circ + (1-\lambda)d/(\lambda+\xi^2),$$
$${}^1/_2\gamma_1 u^\circ + b + {}^1/_2(1-\lambda)x^\circ + (1-\lambda)c/(\lambda+\xi^2),$$
$${}^1/_2\gamma_1 y^\circ + \gamma_1 d/(\lambda+\xi^2),\ {}^1/_2\gamma_2 x^\circ - \gamma_2 c/(\lambda+\xi^2)\}$$
$$H(\theta, \sigma) = {}^1/_2\sigma^2 - G(\theta),\quad G(\theta) = (T_0 - a_1(0)\,\eta^2 - b_1(0))\,\theta -$$
$${}^1/_2 b_2(0)\sin 2\theta + {}^1/_2 b_3(0)\cos 2\theta - \eta c_1(0)\sin\theta + \eta c_2(0)\cos\theta$$

$$l = \frac{1}{\alpha_1 - \gamma_1^2}, \qquad m = \frac{1}{\lambda\alpha_2 - \gamma_2^2}$$

The degenerate system corresponding to (3.3) is conservative and the eigenvalues of the matrix $A(0)$ have negative real parts. Thus, the system (3.3) satisfies the conditions of the theorem given in [11]. According to this theorem the system (3.3) has a periodic solution, if the system

$$\theta^\bullet = \frac{\partial H(\theta\sigma)}{\partial\sigma},\quad \sigma^\bullet = -\frac{\partial H(\theta,\sigma)}{\partial\theta} - \mu\left[Q_2(\theta,0)A^{-1}(0)F(\theta,\sigma)\right] \tag{3.4}$$

also has a periodic solution. Here

$$Q_2(\theta,0)A^{-1}(0)F(\theta,\sigma) = \frac{\sigma}{(\lambda+\xi^3)^3}\{b^2\xi(\xi^4 - 3\lambda\xi^2 + 3\lambda^2\xi^2 - \lambda^3) +$$
$$\frac{1}{2}(\lambda+\xi^2)\left[\frac{\gamma_1^2}{\beta_1}(\lambda^2+\xi^2) + \frac{\gamma_2^2}{\beta_2}(1+\xi^2)\right] + \xi(1-\lambda)^2(\xi^2-\lambda) +$$
$$\frac{1}{2}\cos 2\theta\left[\frac{\gamma_1^2}{\beta_1}(\xi^4 - 3\lambda\xi^2 + 3\lambda^2\xi^2 - \lambda^3) + \frac{\gamma_2^2}{\beta_2}(\lambda - 3\xi^2 + 3\lambda\xi^2 - \xi^4)\right] +$$
$$\sin 2\theta\left[\frac{\gamma_1^2}{\beta_1}\xi(\xi^2+\lambda^3) - \xi\frac{\gamma_2^2}{\beta_2}(1+\lambda\xi^2) + \xi^2(1-\lambda)^3\right] +$$
$$2b\sin\theta\left[-\frac{\gamma_1^2}{\beta_1}\lambda\xi(\lambda^2+\xi^2) + \frac{\gamma_2^2}{\beta_2}\xi^3(\lambda-1) +\right.$$
$$\left.\xi^2(2\lambda - 3\lambda^2 + \lambda^3 + \lambda\xi^2 - \xi^2)\right] + 2b\cos\quad\left[-\frac{\gamma_1^2}{\beta_1}\xi^2(\lambda^2+\xi^2) +\right.$$
$$\left.\frac{\gamma_2^2}{\beta_2}\xi^2(1-\lambda) + \xi(3\lambda\xi^2 - \xi^2 - 2\lambda^2\xi^2 + \lambda^3 - \lambda^2)\right]\}$$

The system (3.4) is nearly a Hamiltonian one. The values of the constants corresponding to the closed curves of the Hamiltonian system $(\mu = 0)$ near which the limit cycles exist for small $\mu \neq 0$, are given by the equation

$$\Psi(h) = \iint \frac{1}{\sigma}[-Q_2(\theta, 0)A^{-1}(0)F(\theta, \sigma)]\,d\theta d\sigma = 0 \qquad (3.5)$$

where the double integral is computed over the area bounded by the closed curve of the conservative system embracing the state of equilibrium [12].

Thus, when $\mu \neq 0$, the system (1.1) has periodic solutions near the closed generator curves $H(\theta, \sigma) = h$ of the degenerate conservative system when the equation (3.5) has the roots $h = h_i$. The periodic solution is stable when $\Psi_h'(h_i) < 0$, and a saddle one when $\Psi_h'(h_i) > 0$. The stable limit cycle corresponds to self-oscillations of the rotor, and its emergence from the composite state of equilibrium corresponds to the "soft" mode of excitation of the self-oscillations [13]. We have

$$\Psi(h) = -2\int_{\varphi_1}^{\varphi_2} \frac{1}{\sigma}[Q_2A^{-1}F]\{2[h + G(\theta)]\}^{1/2}d\theta$$

$$\Psi'(h) = -2\int_{\varphi_1}^{\varphi_2} \frac{1}{\sigma}[Q_2A^{-1}F]\{2[h + G(\theta)]\}^{-1/2}d\theta$$

where φ_1 and φ_2 are the roots of the equation $h + G(\theta) = 0$.

As an example, let us consider a synchronous motor with the same parameters as in Sect. 2. Figures 1, 2 and 3 depict the graphs of the function $\Psi(h)$ versus the parameter ξ, and the correlation between the number n of the curves and the value of ξ is as follows:

$n = 1$	2	3	4	5	6	7	
$\xi = 0$	0.01	0.012	0.0125	0.015	0.01625	0.02	
$n = 8$	9	10	11	12	13	14	15
$\xi = 0.7$	0.74	0.77	0.795	0.7975	0.79875	0.799375	0.8

When $\xi = 0$, the system (1.1) has no limit cycles. At $\xi > 0.012$ a unique saddle-type limit cycle $(\Psi_h'(h_i) > 0)$ appears. It decreases with increasing ξ and merges with the composite focus at $\xi = 0.01625$. When ξ is increased further, the function $\Psi(h)$ appears above the h-axis, and again we have no limit cycles. At $\xi > 0.77$ a stable limit cycle $(\Psi_h'(h_i) < 0)$ appears which decreases with increasing ξ and merges with the composite focus at $\xi = 0.7975$ (the manner in which the cycle appears is not investigated). At $\xi > 0.7975$ the function $\Psi(h)$ appears below the h-axis and again there are no limit cycles. Thus the stable limit cycle corresponding to the self-oscillations of the rotor exists at $0.77 < \xi \leqslant 0.7975$ and the mode of excitation of self-oscillations at $\xi = 0.7975$ is soft.

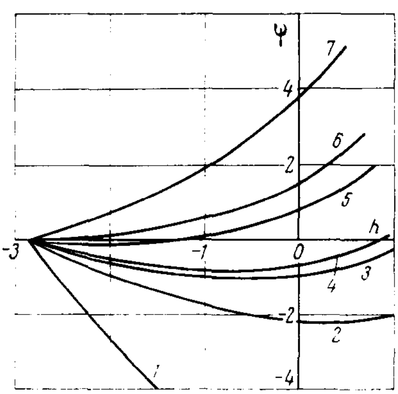

Fig. 1

Let us now consider the synchronous motor in question and inspect the change in the stability of the equilibrium state corresponding to the synchronous mode, with respect to the parameter ξ, using the second order system of equations

$$\theta^{\cdot} = s, \qquad s^{\cdot} = T_0 - M \tag{3.6}$$

as it was done in [14], we obtain the following results. When $\xi = 0$, the state of equilibrium is stable, it becomes unstable when $\xi = 0.01635$ and a unique unstable limit cycle shrinks to it (this unstable limit cycle corresponds to a saddle-type cycle for (1.1)), then at $\xi = 0.7993$ the state of equilibrium becomes again stable and a unique stable limit cycle shrinks to this state. Thus, investigating the emergence of the limit

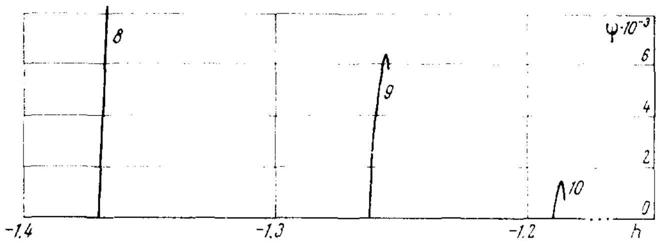

Fig. 2

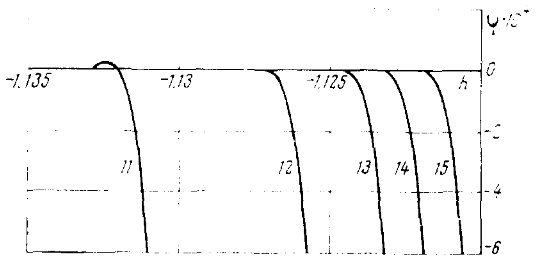

Fig. 3

cycles from the composite state of equilibrium and, in particular, the appearance of self-oscillations, on the basis of the complete system of equations of the dynamics of a synchronous motor yields, in the case of small s, a result which is near (in the above examples the results diverge by less than 1%) to that obtained by considering the second order system of equations (3.6). This confirms once again that for small s the dynamics of a synchronous motor (in particular the swinging of the rotor) can be fully described by a system of second order differential equations.

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SIMULATION OF A VISCOUS COMPRESSIBLE MULTICONSTITUENT FLUID WITH ALLOWANCE FOR POLARIZATION AND MAGNETIZATION EFFECTS

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A variational equation is used for deriving a closed system of equations for defining the behavior of a viscous compressible multiconstituent fluid [1]. The determining parameters comprise besides density, entropy, and mass concentration of constituents, also the polarization and magnetization vectors of individual constituents. In conformity with the method developed in [2 – 4] the mixture is considered to be a single continuous medium so that the presence of various constituents results in additional degrees of internal freedom in the definition of the considered medium. Chemical reactions between mixture constituents and phase transitions are assumed to be absent (*).

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*) The simulation of a viscous multiconstituent fluid with allowance for diffusion and

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